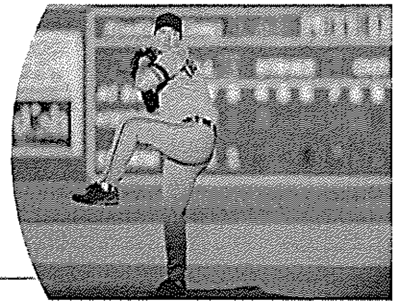
2.1 Use Inductive Reasoning



Before You classified polygons by the number of sides.
Now You will describe patterns and use inductive reasoning.
Why? So you can make predictions about baseball, as in Ex. 32.

Key Vocabulary
- conjecture
- inductive reasoning
- counterexample

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

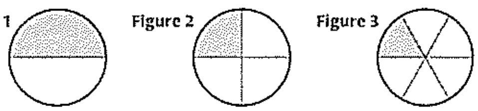


Figure 1 Figure 2 Figure 3

Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

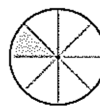


Figure 4

EXAMPLE 2 Describe a number pattern

READ SYMBOLS
The three dots (. . .) tell you that the pattern continues.

Describe the pattern in the numbers −7, −21, −63, −189, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$-7, \xrightarrow{\times 3} -21, \xrightarrow{\times 3} -63, \xrightarrow{\times 3} -189, \ldots \xrightarrow{\times 3}$$

▸ Continue the pattern. The next three numbers are −567, −1701, and −5103.

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✓ **GUIDED PRACTICE** for Examples 1 and 2

1. Sketch the fifth figure in the pattern in Example 1.
2. *Describe* the pattern in the numbers 5.01, 5.03, 5.05, 5.07, ... Write the next three numbers in the pattern.

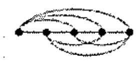
INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?
	$+1$	$+2$	$+3$	$+?$	

▶ **Conjecture** You can connect five collinear points $6 + 4$, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive numbers*. Make and test a conjecture about the sum of any three consecutive numbers.

Solution

STEP 1 **Find** a pattern using a few groups of small numbers.

$3 + 4 + 5 = 12 = 4 \cdot 3$ $\qquad$ $7 + 8 + 9 = 24 = 8 \cdot 3$

$10 + 11 + 12 = 33 = 11 \cdot 3$ $\qquad$ $16 + 17 + 18 = 51 = 17 \cdot 3$

▶ **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 **Test** your conjecture using other numbers. For example, test that it works with the groups $-1, 0, 1$ and $100, 101, 102$.

$-1 + 0 + 1 = 0 = 0 \cdot 3$ ✓ $\qquad$ $100 + 101 + 102 = 303 = 101 \cdot 3$ ✓

✓ **GUIDED PRACTICE** for Examples 3 and 4

3. Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points.
4. Make and test a conjecture about the sign of the product of any three negative integers.

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$-2 + -3 = -5$

$-5 \not> -3$

▶ Because a counterexample exists, the conjecture is false.

★ EXAMPLE 6 Standardized Test Practice

Which conjecture could a high school athletic director make based on the graph at the right?

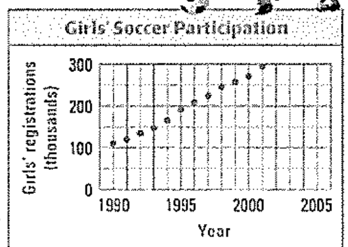
ELIMINATE CHOICES Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.

(A) More boys play soccer than girls.

(B) More girls are playing soccer today than in 1995.

(C) More people are playing soccer today than in the past because the 1994 World Cup games were held in the United States.

(D) The number of girls playing soccer was more in 1995 than in 2001.



Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase from 1990–2001, but does not give any reasons for that increase.

▶ The correct answer is B. (A) (B) (C) (D)

✓ GUIDED PRACTICE for Examples 5 and 6

5. Find a counterexample to show that the following conjecture is false.
   Conjecture The value of x^2 is always greater than the value of x.

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

2.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 19, 22, and 36
◈ = **MULTIPLE REPRESENTATIONS** Ex. 35

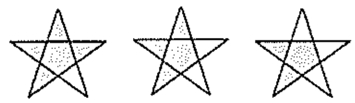
SKILL PRACTICE

1. **VOCABULARY** Write a definition of *conjecture* in your own words.

2. ★ **WRITING** The word *counter* has several meanings. Look up the word in a dictionary. Identify which meaning helps you understand the definition of *counterexample*.

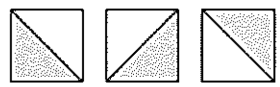
EXAMPLE 1 on p. 72 for Exs. 3–5

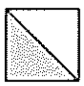
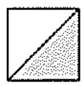
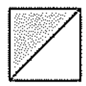
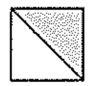
SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern.

3. 

4. 

5. ★ **MULTIPLE CHOICE** What is the next figure in the pattern?



Ⓐ  Ⓑ  Ⓒ  Ⓓ 

EXAMPLE 2 on p. 72 for Exs. 6–11

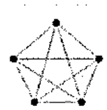
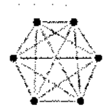
DESCRIBING NUMBER PATTERNS ***Describe*** **the pattern in the numbers. Write the next number in the pattern.**

6. 1, 5, 9, 13, . . .
7. 3, 12, 48, 192, . . .
8. 10, 5, 2.5, 1.25, . . .
9. 4, 3, 1, −2, . . .
10. $1, \frac{2}{3}, \frac{1}{3}, 0, \ldots$
11. −5, −2, 4, 13, . . .

MAKING CONJECTURES **In Exercises 12 and 13, copy and complete the conjecture based on the pattern you observe in the specific cases.**

EXAMPLE 3 on p. 73 for Ex. 12

12. Given seven noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Number of points	3	4	5	6	7
Picture					?
Number of connections	3	6	10	15	?

Conjecture You can connect seven noncollinear points __?__ different ways.

EXAMPLE 4 on p. 73 for Ex. 13

13. Use these sums of odd integers: 3 + 7 = 10, 1 + 7 = 8, 17 + 21 = 38

Conjecture The sum of any two odd integers is __?__.

EXAMPLE 5
on p. 74
for Exs. 14–17

FINDING COUNTEREXAMPLES In Exercises 14–17, show the conjecture is false by finding a counterexample.

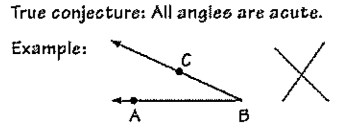
14. If the product of two numbers is positive, then the two numbers must both be positive.

15. The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$.

16. All prime numbers are odd.

17. If the product of two numbers is even, then the two numbers must both be even.

18. ERROR ANALYSIS *Describe* and correct the error in the student's reasoning.



19. ★ SHORT RESPONSE *Explain* why only one counterexample is necessary to show that a conjecture is false.

ALGEBRA In Exercises 20 and 21, write a function rule relating x and y.

20.

x	1	2	3
y	−3	−2	−1

21.

x	1	2	3
y	2	4	6

22. ★ MULTIPLE CHOICE What is the first number in the pattern?

_?_, _?_, _?_, 81, 243, 729

Ⓐ 1 Ⓑ 3 Ⓒ 9 Ⓓ 27

MAKING PREDICTIONS *Describe* a pattern in the numbers. Write the next number in the pattern. Graph the pattern on a number line.

23. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

24. 1, 8, 27, 64, 125, . . .

25. 0.45, 0.7, 0.95, 1.2, . . .

26. 1, 3, 6, 10, 15, . . .

27. 2, 20, 10, 100, 50, . . .

28. $0.4(6), 0.4(6)^2, 0.4(6)^3, \ldots$

29. ALGEBRA Consider the pattern $5, 5r, 5r^2, 5r^3, \ldots$. For what values of r will the values of the numbers in the pattern be increasing? For what values of r will the values of the numbers be decreasing? *Explain.*

30. REASONING A student claims that the next number in the pattern 1, 2, 4, . . . is 8, because each number shown is two times the previous number. Is there another description of the pattern that will give the same first three numbers but will lead to a different pattern? *Explain.*

31. CHALLENGE Consider the pattern $1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, \ldots$.

a. *Describe* the pattern. Write the next three numbers in the pattern.

b. What is happening to the values of the numbers?

c. Make a conjecture about later numbers. *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◈ = MULTIPLE REPRESENTATIONS

2.2 Analyze Conditional Statements

Before You used definitions.
Now You will write definitions as conditional statements.
Why? So you can verify statements, as in Example 2.

Key Vocabulary
- **conditional statement** converse, inverse, contrapositive
- **if-then form** hypothesis, conclusion
- **negation**
- **equivalent statements**
- **perpendicular lines**
- **biconditional statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

$\underbrace{\text{If it is raining,}}_{\text{Hypothesis}}$ then $\underbrace{\text{there are clouds in the sky.}}_{\text{Conclusion}}$

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. Two angles are supplementary if they are a linear pair.

Solution

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

a. All birds have feathers.
If an animal is a bird, then it has feathers.

b. Two angles are supplementary if they are a linear pair.
If two angles are a linear pair, then they are supplementary.

✓ **GUIDED PRACTICE** for Example 1

Rewrite the conditional statement in if-then form.

1. All 90° angles are right angles.
2. $2x + 7 = 1$, because $x = -3$.
3. When $n = 9$, $n^2 = 81$.
4. Tourists at the Alamo are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.
Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.
Negation 2 The cat is black.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the hypothesis and conclusion.

READ VOCABULARY
To *negate* part of a conditional statement, you write its negation.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99°$, then $\angle A$ is obtuse.	both true
Converse If $\angle A$ is obtuse, then $m\angle A = 99°$.	both false
Inverse If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.	both false
Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.	both true

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician.
True, guitars players are musicians.

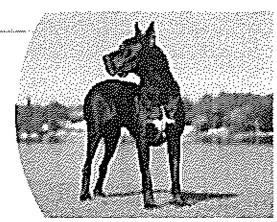
Converse If you are a musician, then you are a guitar player.
False, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician.
False, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

GUIDED PRACTICE for Example 2

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.



5. If a dog is a Great Dane, then it is large.
6. If a polygon is equilateral, then the polygon is regular.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

KEY CONCEPT *For Your Notebook*

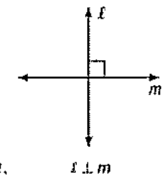
Perpendicular Lines

READ DIAGRAMS
In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



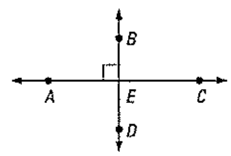
EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.



Solution

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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GUIDED PRACTICE for Example 3

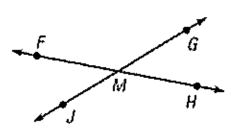
Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.

7. $\angle JMF$ and $\angle FMG$ are supplementary.

8. Point M is the midpoint of $\overline{FH}$.

9. $\angle JMF$ and $\angle HMG$ are vertical angles.

10. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$



READ DEFINITIONS
All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition If two lines intersect to form a right angle, then they are perpendicular.

Converse If two lines are perpendicular, then they intersect to form a right angle.

Biconditional Two lines are perpendicular if and only if they intersect to form a right angle.

✓ **GUIDED PRACTICE** for Example 4

11. Rewrite the definition of *right angle* as a biconditional statement.
12. Rewrite the statements as a biconditional.

    If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

2.2 EXERCISES

HOMEWORK KEY
○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 11, 17, and 33
★ = STANDARDIZED TEST PRACTICE Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The _?_ of a conditional statement is found by switching the hypothesis and the conclusion.
2. ★ **WRITING** Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional.

EXAMPLE 1 on p. 79 for Exs. 3–6

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

3. When $x = 6$, $x^2 = 36$.
4. The measure of a straight angle is 180°.
5. Only people who are registered are allowed to vote.
6. **ERROR ANALYSIS** *Describe* and correct the error in writing the if-then statement.

   Given statement: All high school students take four English courses.

   If-then statement: If a high school student takes four courses, then all four are English courses.

   ✗

EXAMPLE 2
on p. 80
for Exs. 7–15

WRITING RELATED STATEMENTS For the given statement, write the if-then form, the converse, the inverse, and the contrapositive.

7. The complementary angles add to 90°.

8. Ants are insects.

9. $3x + 10 = 16$, because $x = 2$.

10. A midpoint bisects a segment.

ANALYZING STATEMENTS Decide whether the statement is *true* or *false*. If false, provide a counterexample.

11. If a polygon has five sides, then it is a regular pentagon.

12. If $m\angle A$ is 85°, then the measure of the complement of $\angle A$ is 5°.

13. Supplementary angles are always linear pairs.

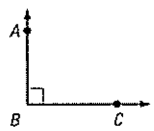
14. If a number is an integer, then it is rational.

15. If a number is a real number, then it is irrational.

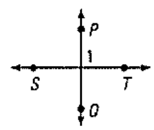
EXAMPLE 3
on p. 81
for Exs. 16–18

USING DEFINITIONS Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.

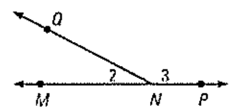
16. $m\angle ABC = 90°$



17. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



18. $m\angle 2 + m\angle 3 = 180°$



EXAMPLE 4
on p. 82
for Exs. 19–21

REWRITING STATEMENTS In Exercises 19–21, rewrite the definition as a biconditional statement.

19. An angle with a measure between 90° and 180° is called *obtuse*.

20. Two angles are a *linear pair* if they are adjacent angles whose noncommon sides are opposite rays.

21. *Coplanar points* are points that lie in the same plane.

DEFINITIONS Determine whether the statement is a valid definition.

22. If two rays are *opposite rays*, then they have a common endpoint.

23. If the sides of a triangle are all the same length, then the triangle is *equilateral*.

24. If an angle is a *right angle*, then its measure is greater than that of an acute angle.

25. ★ **MULTIPLE CHOICE** Which statement has the same meaning as the given statement?

GIVEN ▶ You can go to the movie after you do your homework.

Ⓐ If you do your homework, then you can go to the movie afterwards.

Ⓑ If you do not do your homework, then you can go to the movie afterwards.

Ⓒ If you cannot go to the movie afterwards, then do your homework.

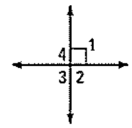
Ⓓ If you are going to the movie afterwards, then do not do your homework.

ALGEBRA **Write the converse of each true statement. Tell whether the converse is true. If false, *explain* why.**

26. If $x > 4$, then $x > 0$.
27. If $x < 6$, then $-x > -6$.
28. If $x \le -x$, then $x \le 0$.

29. ★ **OPEN-ENDED MATH** Write a statement that is true but whose converse is false.

30. **CHALLENGE** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90°$. Use the definition of linear pairs.



PROBLEM SOLVING

EXAMPLE 1 on p. 82 for Exs. 31–32

In Exercises 31 and 32, use the information about volcanoes to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

VOLCANOES Solid fragments are sometimes ejected from volcanoes during an eruption. The fragments are classified by size, as shown in the table.

31. A fragment is called a *block or bomb* if and only if its diameter is greater than 64 millimeters.

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32. A fragment is called a *lapilli* if and only if its diameter is less than 64 millimeters.

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Type of fragment	Diameter d (millimeters)
Ash	$d < 2$
Lapilli	$2 \le d \le 64$
Block or bomb	$d > 64$

33. ★ **SHORT RESPONSE** How can you show that the statement, "If you play a sport, then you wear a helmet." is false? *Explain.*

34. ★ **EXTENDED RESPONSE** You measure the heights of your classmates to get a data set.

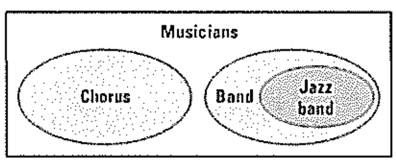
    a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y. *Explain* your reasoning.

    b. Write the converse of the statement in part (a). Is the converse true? *Explain.*

    c. Copy and complete the statement using *mean*, *median*, or *mode* to make a conditional that is true for any data set. *Explain* your reasoning.

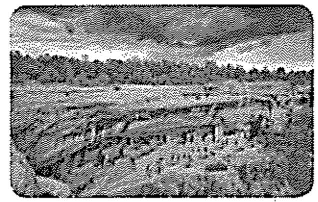
    Statement If a data set has a mean, a median, and a mode, then the __?__ of the data set will always be one of the measurements.

35. ★ **OPEN-ENDED MATH** The Venn diagram below represents all of the musicians at a high school. Write an if-then statement that describes a relationship between the various groups of musicians.



○ = WORKED-OUT SOLUTIONS on p. WS1
★ = STANDARDIZED TEST PRACTICE

2.3 Apply Deductive Reasoning



Before You used inductive reasoning to form a conjecture.

Now You will use deductive reasoning to form a logical argument.

Why So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary
- **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

KEY CONCEPT *For Your Notebook*

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis p, then conclusion q. ⟩ If these statements are true,
<u>If hypothesis q, then conclusion r.</u>
If hypothesis p, then conclusion r. ⟵ then this statement is true.

READ VOCABULARY
The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $BC = XY$.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

a. Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."

"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner.
If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$.
If $x > 5$, then $x^2 > 25$.

c. If a polygon is regular, then all angles in the interior of the polygon are congruent.
If a polygon is regular, then all of its sides are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.
If Rick takes chemistry this year, then Rick will get an A in chemistry.

AVOID ERRORS
The order in which the statements are given does not affect whether you can use the Law of Syllogism.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.
If $x > 5$, then $x^2 > 20$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

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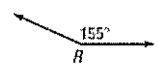
✓ GUIDED PRACTICE for Examples 1 and 2

1. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?



2. If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make?

State the law of logic that is illustrated.

3. If you get an A or better on your math test, then you can go to the movies. If you go to the movies, then you can watch your favorite actor.
If you get an A or better on your math test, then you can watch your favorite actor.

4. If $x > 12$, then $x + 9 > 20$. The value of x is 14.
Therefore, $x + 9 > 20$.

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$, $(-1)(2) = -2$, $2(2) = 4$, $3(2) = 6$,

$(-2)(-4) = 8$, $(-1)(-4) = 4$, $2(-4) = -8$, $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

STEP 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

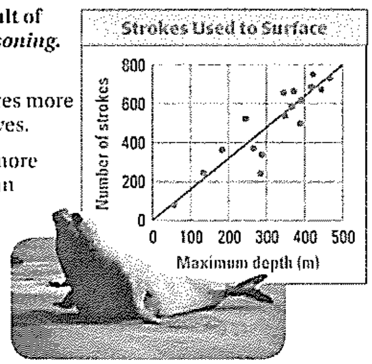
▶ The product of an even integer and any integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your choice.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 60 feet than from 250 feet.



Solution

a. Inductive reasoning, because it is based on a pattern in the data

b. Deductive reasoning, because you are comparing values that are given on the graph

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true.

6. Use inductive reasoning to write another statement about the graph in Example 4. Then use deductive reasoning to write another statement.

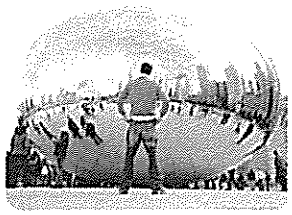
2.3 EXERCISES

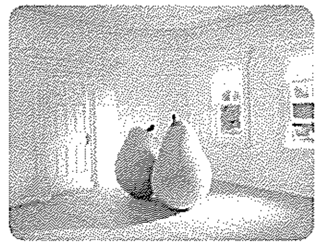
HOMEWORK KEY
○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 7, 17, and 21
★ = STANDARDIZED TEST PRACTICE Exs. 2, 3, 12, 20, and 23

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of _?_.

★ **WRITING** **Use deductive reasoning to make a statement about the picture.**

2. 

3. 

EXAMPLE 1
on p. 87
for Exs. 4–6

LAW OF DETACHMENT **Make a valid conclusion in the situation.**

4. If the measure of an angle is 90°, then it is a right angle. The measure of $\angle A$ is 90°.

5. If $x > 12$, then $-x < -12$. The value of x is 15.

6. If a book is a biography, then it is nonfiction. You are reading a biography.

EXAMPLE 2
on p. 88
for Exs. 7–10

LAW OF SYLLOGISM **In Exercises 7–10, write the statement that follows from the pair of statements that are given.**

7. If a rectangle has four equal side lengths, then it is a square. If a polygon is a square, then it is a regular polygon.

8. If $y > 0$, then $2y > 0$. If $2y > 0$, then $2y - 5 \neq -5$.

9. If you play the clarinet, then you play a woodwind instrument. If you play a woodwind instrument, then you are a musician.

10. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.

EXAMPLE 3
on p. 89
for Ex. 11

11. **REASONING** What can you say about the sum of an even integer and an even integer? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

12. ★ **MULTIPLE CHOICE** If two angles are vertical angles, then they have the same measure. You know that $\angle A$ and $\angle B$ are vertical angles. Using the Law of Detachment, which conclusion could you make?

Ⓐ $m\angle A > m\angle B$
Ⓑ $m\angle A = m\angle B$
Ⓒ $m\angle A + m\angle B = 90°$
Ⓓ $m\angle A + m\angle B = 180°$

13. **ERROR ANALYSIS** *Describe* and correct the error in the argument: "If two angles are a linear pair, then they are supplementary. Angles C and D are supplementary, so the angles are a linear pair."

Extension

Use after Lesson 2.3

Symbolic Notation and Truth Tables

GOAL Use symbolic notation to represent logical statements.

Key Vocabulary
- truth value
- truth table

Conditional statements can be written using *symbolic notation*, where letters are used to represent statements. An arrow ($\rightarrow$), read "implies," connects the hypothesis and conclusion. To write the negation of a statement p you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

KEY CONCEPT — *For Your Notebook*

Symbolic Notation

Let p be "the angle is a right angle" and let q be "the measure of the angle is 90°."

Conditional	If p, then q.	$p \rightarrow q$

Example: If an angle is a right angle, then its measure is 90°.

Converse	If q, then p.	$q \rightarrow p$

Example: If the measure of an angle is 90°, then the angle is a right angle.

Inverse	If not p, then not q.	$\sim p \rightarrow \sim q$

Example: If an angle is not a right angle, then its measure is not 90°.

Contrapositive	If not q, then not p.	$\sim q \rightarrow \sim p$

If the measure of an angle is not 90°, then the angle is not a right angle.

Biconditional	p if and only if q	$p \leftrightarrow q$

Example: An angle is a right angle if and only if its measure is 90°.

EXAMPLE 1 Use symbolic notation

Let p be "the car is running" and let q be "the key is in the ignition."

a. Write the conditional statement $p \rightarrow q$ in words.

b. Write the converse $q \rightarrow p$ in words.

c. Write the inverse $\sim p \rightarrow \sim q$ in words.

d. Write the contrapositive $\sim q \rightarrow \sim p$ in words.

Solution

a. Conditional: If the car is running, then the key is in the ignition.

b. Converse: If the key is in the ignition, then the car is running.

c. Inverse: If the car is not running, then the key is not in the ignition.

d. Contrapositive: If the key is not in the ignition, then the car is not running.

2.5 Reason Using Properties from Algebra

Before You used deductive reasoning to form logical arguments.
Now You will use algebraic properties in logical arguments too.
Why So you can apply a heart rate formula, as in Example 3.

Key Vocabulary
- equation, *p. 875*
- solve an equation, *p. 875*

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT *For Your Notebook*

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	Addition Property of Equality
$5x + 5 = 20$	Combine like terms.	Simplify.
$5x = 15$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 3$	Divide each side by 5.	Division Property of Equality

▸ The value of x is 3.

KEY CONCEPT *For Your Notebook*

Distributive Property

$a(b + c) = ab + ac$, where a, b, and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44.	Division Property of Equality

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EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a.

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70.	Division Property of Equality

GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1. $4x + 9 = -3x + 2$
2. $14x + 3(7 - x) = -1$
3. Solve the formula $A = \frac{1}{2}bh$ for b.

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT *For Your Notebook*

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

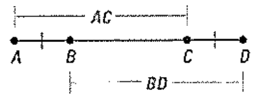
LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.

Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	Transitive Property of Equality

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	Given
$AC = AB + BC$	Add lengths of adjacent segments.	Segment Addition Postulate
$BD = BC + CD$	Add lengths of adjacent segments.	Segment Addition Postulate
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	Addition Property of Equality
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	Substitution Property of Equality

✓ **GUIDED PRACTICE** for Examples 4 and 5

Name the property of equality the statement illustrates.

4. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
5. If $JK = KL$ and $KL = 12$, then $JK = 12$.
6. $m\angle W = m\angle W$

2.5 EXERCISES

HOMEWORK KEY
○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 9, 21, and 31
★ = STANDARDIZED TEST PRACTICE Exs. 2, 5, 27, and 35
◈ = MULTIPLE REPRESENTATIONS Ex. 36

SKILL PRACTICE

1. **VOCABULARY** The following statement is true because of what property? The measure of an angle is equal to itself.

2. ★ **WRITING** *Explain* how to check the answer to Example 3 on page 106.

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 3–14

WRITING REASONS **Copy the logical argument. Write a reason for each step.**

3. $3x - 12 = 7x + 8$ Given
$-4x - 12 = 8$ ___?___
$-4x = 20$ ___?___
$x = -5$ ___?___

4. $5(x - 1) = 4x + 13$ Given
$5x - 5 = 4x + 13$ ___?___
$x - 5 = 13$ ___?___
$x = 18$ ___?___

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.

Ⓐ Substitution Ⓑ Reflexive Ⓒ Symmetric Ⓓ Transitive

WRITING REASONS Solve the equation. Write a reason for each step.

6. $5x - 10 = -40$ 7. $4x + 9 = 16 - 3x$ 8. $5(3x - 20) = -10$

9. $3(2x + 11) = 9$ 10. $2(-x - 5) = 12$ 11. $44 - 2(3x + 4) = -18x$

12. $4(5x - 9) = -2(x + 7)$ 13. $2x - 15 - x = 21 + 10x$ 14. $3(7x - 9) - 19x = -15$

EXAMPLE 3 on p. 106 for Exs. 15–20

ALGEBRA Solve the equation for y. Write a reason for each step.

15. $5x + y = 18$ 16. $-4x + 2y = 8$ 17. $12 - 3y = 30x$

18. $3x + 9y = -7$ 19. $2y + 0.5x = 16$ 20. $\frac{1}{2}x - \frac{3}{4}y = -2$

EXAMPLES 4 and 5 on pp. 107–108 for Exs. 21–25

COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD =$ __?__.
22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then __?__.
23. Addition Property of Equality: If $AB = CD$, then __?__ $+ EF =$ __?__ $+ EF$.
24. Distributive Property: If $5(x + 8) = 2$, then __?__ $x +$ __?__ $= 2$.
25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then __?__.

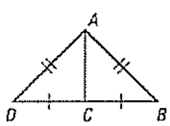
26. **ERROR ANALYSIS** *Describe* and correct the error in solving the equation for x.

$7x = x + 24$	*Given*
$8x = 24$	*Addition Property of Equality*
$x = 3$	*Division Property of Equality*

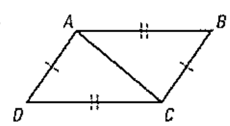
27. ★ **OPEN-ENDED MATH** Write examples from your everyday life that could help you remember the *Reflexive*, *Symmetric*, and *Transitive* Properties of Equality.

PERIMETER In Exercises 28 and 29, show that the perimeter of triangle ABC is equal to the perimeter of triangle ADC.

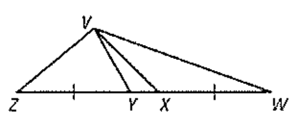
28.



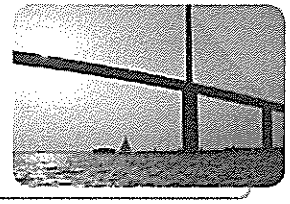
29.



30. **CHALLENGE** In the figure at the right, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW.



2.6 Prove Statements about Segments and Angles



Before You used deductive reasoning.
Now You will write proofs using geometric theorems.
Why? So you can prove angles are congruent, as in Ex. 21.

Key Vocabulary
- proof
- two-column proof
- theorem

A **proof** is a logical argument that shows a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

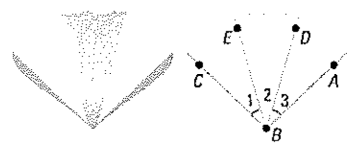
In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

EXAMPLE 1 Write a two-column proof

WRITE PROOFS
Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Write a two-column proof for the situation in Example 4 on page 107.

GIVEN ▶ $m\angle 1 = m\angle 3$
PROVE ▶ $m\angle EBA = m\angle DBC$



STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	4. Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	5. Transitive Property of Equality

GUIDED PRACTICE for Example 1

1. Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN ▶ $AC = AB + AB$
PROVE ▶ $AB = BC$

STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + AB = AB + BC$	3. ?
4. $AB = BC$	4. ?

THEOREMS The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES
Be sure to copy all new theorems in your notebook. Notice that the theorem box tells you where to find the proof(s).

THEOREMS *For Your Notebook*

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs: p. 137; Ex. 5, p. 121; Ex. 26, p. 118

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs: Ex. 25, p. 118; Concept Summary, p. 114; Ex. 21, p. 137

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

Solution

a. Transitive Property of Angle Congruence

b. Symmetric Property of Segment Congruence

✓ **GUIDED PRACTICE** for Example 2

Name the property illustrated by the statement.

2. $\overline{CD} \cong \overline{CD}$

3. If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$.

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

EXAMPLE 3 Use properties of equality

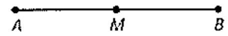
Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

WRITE PROOFS
Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

GIVEN ▶ M is the midpoint of $\overline{AB}$.

PROVE ▶ a. $AB = 2 \cdot AM$

b. $AM = \frac{1}{2}AB$



STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $AM = MB$	3. Definition of congruent segments
4. $AM + MB = AB$	4. Segment Addition Postulate
5. $AM + AM = AB$	5. Substitution Property of Equality
a. 6. $2AM = AB$	6. Distributive Property
b. 7. $AM = \frac{1}{2}AB$	7. Division Property of Equality

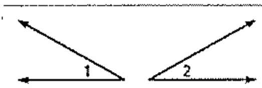
✓ **GUIDED PRACTICE** for Example 3

4. **WHAT IF?** Look back at Example 3. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

CONCEPT SUMMARY *For Your Notebook*

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

Statements based on facts that you know or on conclusions from deductive reasoning

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

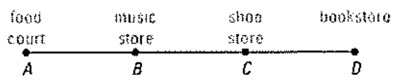
EXAMPLE 4 Solve a multi-step problem

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 120 for the Problem Solving Workshop.

Solution

STEP 1 **Draw** and label a diagram.



STEP 2 **Draw** separate diagrams to show mathematical relationships.

STEP 3 **State** what is given and what is to be proved for the situation. Then write a proof.

GIVEN ▶ B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.

PROVE ▶ $AB = CD$

STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.	1. Given
2. $\overline{AB} \cong \overline{BC}$	2. Definition of midpoint
3. $\overline{BC} \cong \overline{CD}$	3. Definition of midpoint
4. $\overline{AB} \cong \overline{CD}$	4. Transitive Property of Congruence
5. $AB = CD$	5. Definition of congruent segments

✓ **GUIDED PRACTICE** for Example 4

5. In Example 4, does it matter what the actual distances are in order to prove the relationship between AB and CD? *Explain.*

6. In Example 4, there is a clothing store halfway between the music store and the shoe store. What other two store entrances are the same distance from the entrance of the clothing store?

2.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 21
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 4, 12, 19, 27, and 28

SKILL PRACTICE

1. **VOCABULARY** What is a *theorem*? How is it different from a *postulate*?

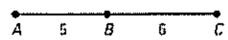
2. ★ **WRITING** You can use theorems as reasons in a two-column proof. What other types of statements can you use as reasons in a two-column proof? Give examples.

EXAMPLE 1 on p. 112 for Exs. 3–4

3. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ $AB = 5$, $BC = 6$

PROVE ▶ $AC = 11$



STATEMENTS	REASONS
1. $AB = 5$, $BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. _?_
4. _?_	4. Simplify.

4. ★ **MULTIPLE CHOICE** Which property listed is the reason for the last step in the proof?

GIVEN ▶ $m\angle 1 = 59°$, $m\angle 2 = 59°$

PROVE ▶ $m\angle 1 = m\angle 2$

STATEMENTS	REASONS
1. $m\angle 1 = 59°$, $m\angle 2 = 59°$	1. Given
2. $59° = m\angle 2$	2. Symmetric Property of Equality
3. $m\angle 1 = m\angle 2$	3. _?_

Ⓐ Transitive Property of Equality Ⓑ Reflexive Property of Equality
Ⓒ Symmetric Property of Equality Ⓓ Distributive Property

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 5–13

USING PROPERTIES **Use the property to copy and complete the statement.**

5. Reflexive Property of Congruence: _?_ $\cong \overline{SE}$

6. Symmetric Property of Congruence: If _?_ $\cong$ _?_, then $\angle RST \cong \angle JKL$.

7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and _?_ $\cong$ _?_, then $\angle F \cong \angle L$.

NAMING PROPERTIES **Name the property illustrated by the statement.**

8. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$.

9. $\angle VWX \cong \angle VWX$

10. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$.

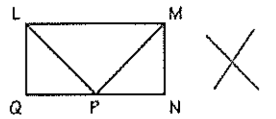
11. $YZ = ZY$

12. ★ **MULTIPLE CHOICE** Name the property illustrated by the statement "If $\overline{CD} \cong \overline{MN}$, then $\overline{MN} \cong \overline{CD}$."

Ⓐ Reflexive Property of Equality Ⓑ Symmetric Property of Equality
Ⓒ Symmetric Property of Congruence Ⓓ Transitive Property of Congruence

13. **ERROR ANALYSIS** In the diagram below, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. *Describe* and correct the error in the reasoning.

Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence.



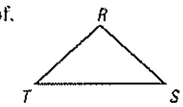
EXAMPLE 4 on p. 115 for Exs. 14–15

MAKING A SKETCH In Exercises 14 and 15, sketch a diagram that represents the given information.

14. **CRYSTALS** The shape of a crystal can be represented by intersecting lines and planes. Suppose a crystal is *cubic*, which means it can be represented by six planes that intersect at right angles.

15. **BEACH VACATION** You are on vacation at the beach. Along the boardwalk, the bike rentals are halfway between your cottage and the kite shop. The snack shop is halfway between your cottage and the bike rentals. The arcade is halfway between the bike rentals and the kite shop.

16. **DEVELOPING PROOF** Copy and complete the proof.



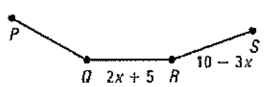
GIVEN ▶ $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$

PROVE ▶ $\overline{RS} \cong \overline{TS}$

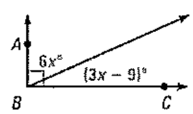
STATEMENTS	REASONS
1. $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$	1. ?
2. $RS = RT$	2. Transitive Property of Equality
3. $RT = TS$	3. Definition of congruent segments
4. $RS = TS$	4. Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	5. ?

ALGEBRA **Solve for x using the given information. *Explain* your steps.**

17. **GIVEN** ▶ $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$



18. **GIVEN** ▶ $m\angle ABC = 90°$



19. ★ **SHORT RESPONSE** *Explain* why writing a proof is an example of deductive reasoning, not inductive reasoning.

20. **CHALLENGE** Point P is the midpoint of $\overline{MN}$ and point Q is the midpoint of $\overline{MP}$. Suppose $\overline{AB}$ is congruent to $\overline{MP}$, and $\overline{PN}$ has length x. Write the length of the segments in terms of x. *Explain.*

a. $\overline{AB}$ b. $\overline{MN}$ c. $\overline{MQ}$ d. $\overline{NQ}$